

Spontaneous Breaking of the BRST Symmetry in the ABJM theory

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In this paper, we will analyse the ghost condensation in the ABJM theory. We will perform our analysis in $\mathcal{N} = 1$ superspace. We show that in the Delbourgo-Jarvis-Baulieu-Thierry-Mieg gauge the spontaneous breaking of BRST symmetry can occur in the ABJM theory. This spontaneous breaking of BRST symmetry is caused by ghost-anti-ghost condensation. We will also show that in the ABJM theory, the ghost-anti-ghost the condensates remains present in the modified abelian gauge. Thus, the spontaneous breaking of BRST symmetry in ABJM theory can even occur in the modified abelian gauge.

I. INTRODUCTION

According to the AdS_4/CFT_3 correspondence the field theory dual to the eleven dimensional supergravity is a superconformal field theory with $\mathcal{N} = 8$ supersymmetry. This is because apart from a constant closed 7-form on S^7 , $AdS_4 \times S^7 \sim [SO(2,3)/SO(1,3)] \times [SO(8)/SO(7)] \subset OSp(8|4)/[SO(1,3) \times SO(7)]$. This group $OSp(8|4)$ gets realized as $\mathcal{N} = 8$ supersymmetry of the dual field theory. Furthermore, the field content of this dual superconformal field theory comprises of eight gauge valued scalar fields and sixteen physical fermions. As this theory only has sixteen on shell degrees of freedom, so, the gauge fields cannot have any contribution to the on shell degrees of freedom. Thus, the gauge sector of this theory is represented by Chern-Simons type actions. A theory called the the Bagger-Lambert-Gustavsson (BLG) theory meets all these requirements [1, 2].

The gauge symmetry in the BLG theory is generated by a Lie 3-algebra rather than a Lie algebra and $SO(4)$ is the only known example of a Lie 3-algebra. It is possible to decompose the gauge symmetry generated by $SO(4)$ into $SU(2) \times SU(2)$. If we do that for the BLG theory, then its gauge symmetry is generated by ordinary Lie algebras. The gauge sector of the BLG theory is now represented by two Chern-Simons theories with levels $\pm k$ and the matter fields exist in the bi-fundamental representation. The BLG theory only represents two M2-branes because its the gauge symmetry is generated by the gauge group $SU(2)_k \times SU(2)_{-k}$. However, it has been possible to extend the gauge group to $U(N)_k \times U(N)_{-k}$, and the resultant theory is called Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [3]. Even though, the ABJM theory only has $\mathcal{N} = 6$ supersymmetry, this supersymmetry gets enhanced to $\mathcal{N} = 8$ supersymmetry for Chern-Simons levels, $k = 1, 2$ [4]. Furthermore, for two M2-branes ABJM theory coincides with the BLG theory and thus has $\mathcal{N} = 8$ supersymmetry.

It may be noted that as the ABJM theory has gauge symmetry, it cannot be quantized without getting rid of these unphysical degrees of freedom. This can be done by fixing a gauge. The gauge fixing condition can be incorporated at a quantum level by adding ghost and gauge fixing terms to the original classical Lagrangian. It is known that for a gauge theory the new effective Lagrangian constructed as the sum

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of the original classical Lagrangian with the gauge fixing and the ghost terms, is invariant a new set of transformations called the Becchi-Rouet-Stora-Tyutin (BRST) transformations [5, 6]. Recently, BRST symmetry has also been studied in non-linear gauges also [7, 8]. The BRST symmetry for the ABJM theory has also been studied [9–13].

The ghost-anti-ghost condensation in gauge theories has been throughly studied [14–18]. Such condensation can lead to a spontaneous breaking of the BRST and the anti-BRST symmetries. In fact, in recent past such ghost-anti-ghost condensation has been proposed as a mechanism of providing the masses of off-diagonal gluons and off-diagonal ghosts in the Yang-Mills theory in the Maximally Abelian gauge [19, 20]. This mechanism helps in providing evidences for the infrared Abelian dominance [21], thereby justifies the dual superconductor picture [22–24] of QCD vacuum for explaining quark confinement [20, 25–27]. It may be noted that breaking of BRST symmetry has led to many interesting consequences [14, 28–31]. Therefore, these give us sufficient motivations to study the spontaneous breaking of the BRST and the anti-BRST symmetries in ABJM theory.

With these motivations, in this paper, we establish the Chern-Simons-matter theory with different labels k and $-k$ in non-linear Delbourgo-Jarvis-Baulieu-Thierry-Mieg (DJBTM) gauge. Further, the quantum actions generated due to these gauge conditions are shown to posses supersymmetric BRST and anti-BRST invariance. A novel feature of ABJM theory in non-linear DJBTM gauge is observed that the presence of ghost-anti-ghost condensates in the theory breaks both the BRST and anti-BRST symmetries spontaneously. In accordance with Nambu-Goldstone theorem the Faddeev-Popov ghost and anti-ghost fields of the ABJM theory are identified as Nambu-Goldstone particles. Furthermore, we construct the effective potentials in case ABJM theory explicitly which confirm the appearance of ghost-anti-ghost condensation as well as spontaneous symmetry breaking in the theory. Finally, we will analyse the ghost-anti-ghost condensation in the ABJM theory in modified maximally abelian (MA) gauge.

II. THE ABJM THEORY IN $\mathcal{N} = 1$ SUPERSPACE

In this section we will review the construction of Lagrangian for ABJM theory in $\mathcal{N} = 1$ superspace formalism. The three dimensional $\mathcal{N} = 1$ superspace is parameterized by three spacetime coordinates along with a two component anti-commuting parameter, θ^a . Now $Q_a = \partial_a - (\gamma^\mu \partial_\mu)_a^b \theta_b$, is the generator of $\mathcal{N} = 1$ supersymmetry. This generator of $\mathcal{N} = 1$ supersymmetry commutes with a superspace derivative, $D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b$. This superspace derivative can be used to construct the Lagrangian for ABJM theory in $\mathcal{N} = 1$ superspace formalism. As the ABJM theory is a gauge theory with the gauge group $U(N)_k \times U(N)_{-k}$, we can write the Lagrangian for ABJM theory as

$$\mathcal{L}_c = \mathcal{L}_M + \mathcal{L}_{CS} - \tilde{\mathcal{L}}_{CS}, \quad (1)$$

where $\mathcal{L}_{CS}, \tilde{\mathcal{L}}_{CS}$ are Chern-Simons Lagrangians, and \mathcal{L}_M is Lagrangian for the matter fields. These Chern-Simons Lagrangian are defined by

$$\begin{aligned} \mathcal{L}_{CS} &= \frac{k}{2\pi} \int d^2\theta \, Tr [\Gamma^a \Omega_a], \\ \tilde{\mathcal{L}}_{CS} &= \frac{k}{2\pi} \int d^2\theta \, Tr [\tilde{\Gamma}^a \tilde{\Omega}_a], \end{aligned} \quad (2)$$

where k is an integer and

$$\Omega_a = \frac{1}{2} D^b D_a \Gamma_b - \frac{i}{2} [\Gamma^b, D_b \Gamma_a] - \frac{1}{6} [\Gamma^b, \{\Gamma_b, \Gamma_a\}] - \frac{1}{6} [\Gamma^b, \Gamma_{ab}], \quad (3)$$

$$\begin{aligned} \Gamma_{ab} &= -\frac{i}{2} [D_{(a} \Gamma_{b)} - i \{\Gamma_a, \Gamma_b\}], \\ \tilde{\Omega}_a &= \frac{1}{2} D^b D_a \tilde{\Gamma}_b - \frac{i}{2} [\tilde{\Gamma}^b, D_b \tilde{\Gamma}_a] - \frac{1}{6} [\tilde{\Gamma}^b, \{\tilde{\Gamma}_b, \tilde{\Gamma}_a\}] - \frac{1}{6} [\tilde{\Gamma}^b, \tilde{\Gamma}_{ab}], \end{aligned} \quad (4)$$

$$\tilde{\Gamma}_{ab} = -\frac{i}{2} [D_{(a} \tilde{\Gamma}_{b)} - i \{\tilde{\Gamma}_a, \tilde{\Gamma}_b\}]. \quad (5)$$

The fields Γ_a and $\tilde{\Gamma}_a$ are matrix valued spinor superfields suitable contracted with generator T_A of Lie algebra as $\Gamma_a = \Gamma_a^A T_A$ and $\tilde{\Gamma}_a = \tilde{\Gamma}_a^A T_A$, respectively and they are expressed in component form as

$$\begin{aligned}\Gamma_a &= \chi_a + B\theta_a + \frac{1}{2}(\gamma^\mu)_a A_\mu + i\theta^2 \left[\lambda_a - \frac{1}{2}(\gamma^\mu \partial_\mu \chi)_a \right], \\ \tilde{\Gamma}_a &= \tilde{\chi}_a + \tilde{B}\theta_a + \frac{1}{2}(\gamma^\mu)_a \tilde{A}_\mu + i\theta^2 \left[\tilde{\lambda}_a - \frac{1}{2}(\gamma^\mu \partial_\mu \tilde{\chi})_a \right].\end{aligned}\quad (6)$$

Thus, in component form these Lagrangian are given by

$$\begin{aligned}\mathcal{L}_{cs} &= \frac{k}{4\pi} \left(2\epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \frac{4i}{3} A_\mu A_\nu A_\rho + E^a E_a + \mathcal{D}_\mu (\chi^a (\gamma^\mu)_a^b E_b) \right), \\ \tilde{\mathcal{L}}_{cs} &= \frac{k}{4\pi} \left(2\epsilon^{\mu\nu\rho} \tilde{A}_\mu \partial_\nu \tilde{A}_\rho + \frac{4i}{3} \tilde{A}_\mu \tilde{A}_\nu \tilde{A}_\rho + \tilde{E}^a \tilde{E}_a + \tilde{\mathcal{D}}_\mu (\tilde{\chi}^a (\gamma^\mu)_a^b \tilde{E}_b) \right).\end{aligned}\quad (7)$$

The explicit expression for Lagrangian of the matter fields is given by

$$\mathcal{L}_M = \frac{1}{4} \int d^2 \theta \text{Tr} [\nabla^a X^{I\dagger} \nabla_a X_I + \mathcal{V}], \quad (8)$$

where the super-covariant derivatives for matrix valued complex scalar superfields X^I and $X^{I\dagger}$ are defined by

$$\begin{aligned}\nabla_a X^I &= D_a X^I + i\Gamma_a X^I - iX^I \tilde{\Gamma}_a, \\ \nabla_a X^{I\dagger} &= D_a X^{I\dagger} - iX^{I\dagger} \Gamma_a + i\tilde{\Gamma}_a X^{I\dagger},\end{aligned}\quad (9)$$

and \mathcal{V} is the potential term given by

$$\mathcal{V} = \frac{16\pi}{k} \epsilon^{IJ} \epsilon_{KL} [X_I X^{K\dagger} X_J X^{L\dagger}]. \quad (10)$$

The classical Lagrangian for ABJM theory \mathcal{L}_c remains invariant under the following gauge transformation

$$\begin{aligned}\delta\Gamma_a &= \nabla_a \Lambda, \quad \delta\tilde{\Gamma}_a = \tilde{\nabla}_a \tilde{\Lambda}, \\ \delta X^I &= i(\Lambda X^I - X^I \tilde{\Lambda}), \quad \delta X^{I\dagger} = i(\tilde{\Lambda} X^{I\dagger} - X^{I\dagger} \Lambda),\end{aligned}\quad (11)$$

where $\Lambda = \Lambda^A T_A$ and $\tilde{\Lambda} = \tilde{\Lambda}^A \tilde{T}_A$ are parameters of transformations. The Lagrangian for the ABJM theory is invariant under these gauge transformations.

III. ABJM THEORY IN DJBTM GAUGE

The gauge invariance of ABJM theory reflects that the theory is endowed with some spurious degrees of freedom. In order to quantize the theory correctly we need to fix the gauge. In this section, we will analyse the ABJM theory in Delbourgo-Jarvis-Baulieu-Thierry-Mieg (DJBTM) Gauge [32, 33]. We start with proper choices of the covariant gauge fixing conditions for ABJM theory which remove the spurious degrees of freedom as follows $G_1 \equiv D^a \Gamma_a = 0$, $\tilde{G}_1 \equiv D^a \tilde{\Gamma}_a = 0$ [9]. These gauge fixing conditions can be incorporated at the quantum level by adding the following gauge fixing term with gauge parameter α to the original Lagrangian ,

$$\mathcal{L}_{gf} = \int d^2 \theta \text{Tr} \left[b(D^a \Gamma_a) + \frac{\alpha}{2} b\tilde{b} - \tilde{b}(D^a \tilde{\Gamma}_a) - \frac{\alpha}{2} \tilde{b}\tilde{b} \right], \quad (12)$$

where b and \tilde{b} are Nakanishi-Lautrup type auxiliary fields. The Faddeev-Popov ghost terms corresponding to the above gauge fixing term can be written in terms of ghost fields c, \tilde{c} and corresponding anti-ghost fields $\bar{c}, \tilde{\bar{c}}$ explicitly as

$$\mathcal{L}_{gh} = i \int d^2 \theta \text{Tr} [\bar{c} D^a \nabla_a c - \tilde{\bar{c}} D^a \tilde{\nabla}_a \tilde{c}]. \quad (13)$$

These gauge-fixing and ghost terms are BRST exact and defined together by

$$\mathcal{L}_g = \mathcal{L}_{gf} + \mathcal{L}_{gh}. \quad (14)$$

Now, the nilpotent BRST transformations (i.e. $s_b^2 = 0$) are constructed by

$$\begin{aligned} s_b \Gamma_a &= \nabla_a c, & s_b \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{c}, \\ s_b c &= -\frac{1}{2}[c, c], & s_b \tilde{c} &= -\frac{1}{2}[\tilde{c}, \tilde{c}], \\ s_b \bar{c} &= i\bar{b}, & s_b \tilde{\bar{c}} &= i\tilde{\bar{b}}, \\ s_b b &= 0, & s_b \tilde{b} &= 0, \\ s_b X^I &= i\bar{c}X^I - iX^I\tilde{c}, & s_b X^{I\dagger} &= i\tilde{c}X^{I\dagger} - iX^{I\dagger}\bar{c}, \\ s_b Y^I &= i\tilde{c}Y^I - iY^I\bar{c}, & s_b Y^{I\dagger} &= i\bar{c}Y^{I\dagger} - iY^{I\dagger}\tilde{c}, \end{aligned} \quad (15)$$

which leave the effective ABJM Lagrangian $\mathcal{L}_{ABJ} = \mathcal{L}_c + \mathcal{L}_g$ invariant. With the help of this BRST symmetry the gauge fixing and ghost parts of the effective Lagrangian can be expressed as

$$\mathcal{L}_g = is_b \int d^2\theta \text{Tr} \left[\tilde{c}D^a\tilde{\Gamma}_a + \frac{\alpha}{2}\tilde{b} - \bar{c}D^a\Gamma_a - \frac{\alpha}{2}b \right]. \quad (16)$$

Furthermore, we explore the another nilpotent symmetry so-called anti-BRST transformations for ABJM theory, where the role of ghost and anti-ghost fields are interchanged, as follows

$$\begin{aligned} s_{ab} \Gamma_a &= \nabla_a \bar{c}, & s_{ab} \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{\bar{c}}, \\ s_{ab} \bar{c} &= -\frac{1}{2}[\bar{c}, \bar{c}], & s_{ab} \tilde{\bar{c}} &= -\frac{1}{2}[\tilde{\bar{c}}, \tilde{\bar{c}}], \\ s_{ab} c &= i\bar{b}, & s_{ab} \tilde{c} &= i\tilde{\bar{b}}, \\ s_{ab} \bar{b} &= 0, & s_{ab} \tilde{\bar{b}} &= 0, \\ s_{ab} X^I &= i\bar{c}X^I - iX^I\tilde{\bar{c}}, & s_{ab} X^{I\dagger} &= i\tilde{\bar{c}}X^{I\dagger} - iX^{I\dagger}\bar{c}, \\ s_{ab} Y^I &= i\tilde{\bar{c}}Y^I - iY^I\bar{c}, & s_{ab} Y^{I\dagger} &= i\bar{c}Y^{I\dagger} - iY^{I\dagger}\tilde{\bar{c}}. \end{aligned} \quad (17)$$

Here we remark that the newly added auxiliary fields \bar{b} and $\tilde{\bar{b}}$ can be expressed in terms of original auxiliary fields b and \tilde{b} as following:

$$\bar{b} = -b + i[c, \bar{c}], \quad \tilde{\bar{b}} = -\tilde{b} + i[\tilde{c}, \tilde{\bar{c}}]. \quad (18)$$

These conditions are similar to Curci-Ferrari (CF) type restriction.

Now, we will construct the effective Lagrangian for the ABJM theory in DJBTM gauge. For this purpose, we construct the sum of gauge-fixing and ghost parts of the effective Lagrangian in DJBTM gauge which is defined by

$$\begin{aligned} \mathcal{L}_g^{DJ} &= \int d^2\theta \text{Tr} \left[b(D^a\Gamma_a) + \frac{\alpha}{2}bb - \tilde{b}(D^a\tilde{\Gamma}_a) - \frac{\alpha}{2}\tilde{b}\tilde{b} + i\bar{c}D^a\nabla_a c \right. \\ &\quad \left. - i\tilde{c}D^a\tilde{\nabla}_a \tilde{c} - \frac{\alpha}{2}i[c, \bar{c}]b + \frac{\alpha}{8}[\bar{c}, \bar{c}][c, c] + \frac{\alpha}{2}i[\tilde{c}, \tilde{\bar{c}}]\tilde{b} \right. \\ &\quad \left. - \frac{\alpha}{8}[\tilde{\bar{c}}, \tilde{\bar{c}}][\tilde{c}, \tilde{c}] \right]. \end{aligned} \quad (19)$$

This can further be written as

$$\begin{aligned} \mathcal{L}_g^{DJ} &= \int d^2\theta \text{Tr} \left[b(D^a\Gamma_a) + \frac{\alpha}{2}bb - \tilde{b}(D^a\tilde{\Gamma}_a) - \frac{\alpha}{2}\tilde{b}\tilde{b} + i\bar{c}D^a\nabla_a c \right. \\ &\quad \left. - i\tilde{c}D^a\tilde{\nabla}_a \tilde{c} - \frac{\alpha}{2}i[c, \bar{c}]b - \frac{\alpha}{4}[c, \bar{c}][c, \bar{c}] + \frac{\alpha}{2}i[\tilde{c}, \tilde{\bar{c}}]\tilde{b} \right. \\ &\quad \left. + \frac{\alpha}{4}[\tilde{\bar{c}}, \tilde{\bar{c}}][\tilde{c}, \tilde{c}] \right]. \end{aligned} \quad (20)$$

The effective Lagrangian for ABJM theory in DJBTM gauge, $\mathcal{L}_c + \mathcal{L}_g^{DJ}$, is also invariant under above set of BRST and anti-BRST transformations mentioned in Eqs. (15) and (17). Further, we can express the Lagrangian \mathcal{L}_g^{DJ} in terms of both BRST as well as anti-BRST exact term

$$\mathcal{L}_g^{DJ} = \frac{i}{2} s\bar{s} \int d^2\theta \text{Tr}[\Gamma_a \Gamma^a - \tilde{\Gamma}_a \tilde{\Gamma}^a - i\alpha \bar{c}c + i\alpha \tilde{c}\tilde{c}]. \quad (21)$$

Now, to inspect the non-zero gauge parameter, we write the above Lagrangian as

$$\begin{aligned} \mathcal{L}_g^{DJ} = & \int d^2\theta \text{Tr} \left[\frac{\alpha}{2} \left(b - \frac{1}{2}i[c, \bar{c}] + \frac{1}{\alpha} D_a \Gamma^a \right)^2 - \frac{1}{2\alpha} (D_a \Gamma^a)^2 + i\bar{c} D^a \nabla_a c \right. \\ & - \frac{\alpha}{2} \left(\tilde{b} - \frac{1}{2}i[\tilde{c}, \tilde{\bar{c}}] + \frac{1}{\alpha} D_a \tilde{\Gamma}^a \right)^2 + \frac{1}{2\alpha} (D_a \tilde{\Gamma}^a)^2 - i\tilde{c} D^a \tilde{\nabla}_a \tilde{c} \\ & \left. - \frac{\alpha}{8} [c, \bar{c}][c, \bar{c}] + \frac{\alpha}{8} [\tilde{c}, \tilde{\bar{c}}][\tilde{c}, \tilde{\bar{c}}] \right]. \end{aligned} \quad (22)$$

For analysing the spontaneous breaking of BRST symmetry, the non-linear auxiliary fields b and \tilde{b} could play an important role as an order parameters for the BRST symmetry breaking. Therefore, we would not remove them by using equations of motion.

IV. SPONTANEOUS BREAKING OF BRST SYMMETRY

In this section, we describe the breaking of BRST supersymmetry spontaneously in case of ABJM theory. To do so, let us define the potential $V(b, \tilde{b})$ for multiplier fields b and \tilde{b} such that

$$\begin{aligned} V(b, \tilde{b}) = & \int d^2\theta \text{Tr} \left[-\frac{\alpha}{2} \left(b - \frac{1}{2}i[c, \bar{c}] + \frac{1}{\alpha} D_a \Gamma^a \right)^2 \right. \\ & \left. + \frac{\alpha}{2} \left(\tilde{b} - \frac{1}{2}i[\tilde{c}, \tilde{\bar{c}}] + \frac{1}{\alpha} D_a \tilde{\Gamma}^a \right)^2 \right]. \end{aligned} \quad (23)$$

The potential has its extremum for gauge parameter α for any integer value either at

$$b = \frac{1}{2}i[c, \bar{c}] - \frac{1}{\alpha} D_a \Gamma^a \text{ and } \tilde{b} = \frac{1}{2}i[\tilde{c}, \tilde{\bar{c}}] - \frac{1}{\alpha} D_a \tilde{\Gamma}^a. \quad (24)$$

The vacuum states of non-linear bosonic fields b and \tilde{b} are given by

$$\langle 0|b|0\rangle = \frac{1}{2}\langle 0|i[c, \bar{c}]|0\rangle, \quad \langle 0|\tilde{b}|0\rangle = \frac{1}{2}\langle 0|i[\tilde{c}, \tilde{\bar{c}}]|0\rangle, \quad (25)$$

where we have utilized the invariance such that $\langle \Gamma_a \rangle = 0$ and $\langle \tilde{\Gamma}_a \rangle = 0$. In case of ghost-anti-ghost condensations appear such that

$$\langle 0|i[c, \bar{c}]|0\rangle \neq 0, \quad \langle 0|i[\tilde{c}, \tilde{\bar{c}}]|0\rangle \neq 0, \quad (26)$$

the non-linear fields b and \tilde{b} acquire non-vanishing vacuum to vacuum expectation values (VEVs), i.e.,

$$\langle 0|b|0\rangle = \frac{1}{2}\langle 0|i[c, \bar{c}]|0\rangle \neq 0, \quad \langle 0|\tilde{b}|0\rangle = \frac{1}{2}\langle 0|i[\tilde{c}, \tilde{\bar{c}}]|0\rangle \neq 0. \quad (27)$$

Consequently, these non-vanishing VEVs break the BRST symmetry spontaneously as follows:

$$\begin{aligned} \langle 0|s_b \bar{c}|0\rangle &= \langle 0|ib|0\rangle = -\frac{1}{2}\langle 0|[c, \bar{c}]|0\rangle \neq 0, \\ \langle 0|s_b \tilde{c}|0\rangle &= \langle 0|i\tilde{b}|0\rangle = -\frac{1}{2}\langle 0|[\tilde{c}, \tilde{\bar{c}}]|0\rangle \neq 0. \end{aligned} \quad (28)$$

Using CF conditions given in Eq. (18), it is easy to see that spontaneous breaking of anti-BRST symmetry also occurs in this case

$$\begin{aligned}\langle 0|s_{ab}c|0\rangle &= \langle 0|i\bar{b}|0\rangle = -\frac{1}{2}\langle 0|[c, \bar{c}]|0\rangle \neq 0, \\ \langle 0|s_{ab}\tilde{c}|0\rangle &= \langle 0|i\tilde{\bar{b}}|0\rangle = -\frac{1}{2}\langle 0|[\tilde{c}, \tilde{\bar{c}}]|0\rangle \neq 0.\end{aligned}\quad (29)$$

According to Nambu-Goldstone theorem we note that, corresponding to these spontaneous symmetries breaking there exist massless Nambu-Goldstone particles. For instance, these ghosts and anti-ghosts can be identified as Nambu-Goldstone particles.

To determine whether such ghost-anti-ghost condensation as well as spontaneous symmetry breaking take place or not, we express the effective potential in case of ABJM theory as

$$V(b, \tilde{b}, \phi, \tilde{\phi}) = V(\phi, \tilde{\phi}) + \int d^2\theta \operatorname{Tr} \left[-\frac{\alpha}{2} \left(b + \frac{1}{2\alpha} \phi \right)^2 + \frac{\alpha}{2} \left(\tilde{b} + \frac{1}{2\alpha} \tilde{\phi} \right)^2 \right], \quad (30)$$

where $\phi \sim -\alpha\langle 0|i[c, \bar{c}]|0\rangle$ and $\tilde{\phi} \sim -\alpha\langle 0|i[\tilde{c}, \tilde{\bar{c}}]|0\rangle$. However, it can be noticed that for Landau gauge condition where gauge parameter takes zero value such kind of ghost-anti-ghost condensation and consequently spontaneous symmetry breaking does not appear for ABJM theory in $\mathcal{N} = 1$ superspace. This result verifies the conventional use of the BRST symmetry for ABJM theory in the linear gauge.

V. ABJM THEORY IN MA GAUGE

Now we analyse ghost-anti-ghost condensation in modified maximally abelian (MA) gauge [34, 35]. Thus, we start by decomposing the gauge fields in diagonal and off-diagonal components as follows

$$\Gamma_a = \gamma_a^i T_i + \Upsilon_a^\alpha T_\alpha, \quad \tilde{\Gamma}_a = \tilde{\gamma}_a^i \tilde{T}_i + \tilde{\Upsilon}_a^\alpha \tilde{T}_\alpha, \quad (31)$$

where $T_i \in \mathcal{H}$ and $T_\alpha \in U(N)_k - \mathcal{H}$ with \mathcal{H} being the Cartan subalgebra of the Lie algebra $U(N)_k$. Similarly, $\tilde{T}_i \in \mathcal{H}$ and $\tilde{T}_\alpha \in U(N)_{-k} - \mathcal{H}$ with \mathcal{H} being the Cartan subalgebra of the Lie algebra $U(N)_{-k}$. Now, the Lagrangian in MA gauge is constructed in terms of BRST exact quantity

$$\begin{aligned}\mathcal{L}_g^{MA} &= -is \int d^2\theta \operatorname{Tr} \left[\bar{c} \left\{ \nabla_a[\gamma] \Upsilon^a + \frac{\alpha}{2} b \right\} - i \frac{\zeta}{2} \bar{c}[\bar{c}, c] - i \frac{\zeta}{4} c[\bar{c}, \bar{c}] \right. \\ &\quad \left. - \tilde{\bar{c}} \left\{ \tilde{\nabla}_a[\tilde{\gamma}] \tilde{\Upsilon}^a + \frac{\alpha}{2} \tilde{b} \right\} + i \frac{\zeta}{2} \tilde{\bar{c}}[\tilde{\bar{c}}, \tilde{c}] + i \frac{\zeta}{4} \tilde{c}[\tilde{\bar{c}}, \tilde{c}] \right].\end{aligned}\quad (32)$$

Utilizing BRST transformation the above Lagrangian is further expanded as

$$\begin{aligned}\mathcal{L}_g^{MA} &= \int d^2\theta \operatorname{Tr} \left[b \nabla_a[\gamma] \Upsilon^a + \frac{\alpha}{2} b^2 \right. \\ &\quad + i \bar{c} \nabla_a[\gamma] \nabla^a[\gamma] c - i [\bar{c}, \Upsilon_a][c, \Upsilon^a] + i \bar{c} \nabla_a[\gamma] ([\Upsilon^a, c]) \\ &\quad + i \bar{c} [\nabla_a[\gamma] \Upsilon^a, c] + \frac{\zeta}{8} [\bar{c}, \bar{c}][c, c] + \frac{\zeta}{4} [\bar{c}, \bar{c}][c, c] \\ &\quad + i \frac{\zeta}{2} c[b, \bar{c}] - i \zeta b[\bar{c}, c] + \frac{\zeta}{4} [\bar{c}, \bar{c}][c, c] \\ &\quad - \tilde{b} \tilde{\nabla}_a[\tilde{\gamma}] \tilde{\Upsilon}^a - \frac{\alpha}{2} \tilde{b}^2 - i \tilde{\bar{c}} \tilde{\nabla}_a[\tilde{\gamma}] \tilde{\nabla}^a[\tilde{\gamma}] \tilde{c} \\ &\quad + i [\tilde{\bar{c}}, \tilde{\Upsilon}_a][\tilde{c}, \tilde{\Upsilon}^a] - i \tilde{\bar{c}} \tilde{\nabla}_a[\tilde{\gamma}] ([\tilde{\Upsilon}^a, \tilde{c}]) \\ &\quad \left. - i \tilde{\bar{c}} [\tilde{\nabla}_a[\tilde{\gamma}] \tilde{\Upsilon}^a, \tilde{c}] - \frac{\zeta}{8} [\tilde{\bar{c}}, \tilde{\bar{c}}][\tilde{c}, \tilde{c}] \right]\end{aligned}$$

$$\begin{aligned}
& -\frac{\zeta}{4}[\tilde{c}, \tilde{c}][\tilde{c}, \tilde{c}] - i\frac{\zeta}{2}\tilde{c}[\tilde{b}, \tilde{c}] + i\zeta b[\tilde{c}, \tilde{c}] \\
& -\frac{\zeta}{4}[\tilde{c}, \tilde{c}][\tilde{c}, \tilde{c}].
\end{aligned} \tag{33}$$

In the modified MA gauge, the requirement of the orthosymplectic invariance yields the quartic ghost interaction as $\zeta = \alpha$. In the modified MA gauge, the above expression reduces to

$$\begin{aligned}
\mathcal{L}_g^{MMA} = & \int d^2\theta \text{Tr} \left[\frac{\alpha}{2} \left(b - i[\tilde{c}, c] + \frac{1}{\alpha} \nabla_a[\gamma] \Upsilon^a \right)^2 - \frac{1}{2\alpha} (\nabla_a[\gamma] \Upsilon^a)^2 \right. \\
& - i[\tilde{c}, c][\Upsilon_a, \Upsilon^a] + i\tilde{c} \nabla_a[\gamma]([\Upsilon^a, c]) - \frac{\alpha}{2} \left(\tilde{b} - i[\tilde{c}, \tilde{c}] + \frac{1}{\alpha} \tilde{\nabla}_a[\tilde{\gamma}] \tilde{\Upsilon}^a \right)^2 \\
& + i\tilde{c} \tilde{\nabla}_a[\tilde{\gamma}] \tilde{\nabla}^a[\tilde{\gamma}] c + i[\tilde{c}, \tilde{c}][\tilde{\Upsilon}_a, \tilde{\Upsilon}^a] - i\tilde{c} \tilde{\nabla}_a[\tilde{\gamma}]([\tilde{\Upsilon}^a, \tilde{c}]) \\
& \left. - i\tilde{c} \nabla_a[\gamma] \nabla^a[\gamma] c + \frac{1}{2\alpha} (\tilde{\nabla}_a[\tilde{\gamma}] \tilde{\Upsilon}^a)^2 \right].
\end{aligned} \tag{34}$$

The potential for non-linear field b and \tilde{b} has extremum either at

$$\begin{aligned}
b &= i[\tilde{c}, c] - \frac{1}{\alpha} \nabla_a[\gamma] \Upsilon^a, \\
\tilde{b} &= i[\tilde{c}, \tilde{c}] - \frac{1}{\alpha} \tilde{\nabla}_a[\tilde{\gamma}] \tilde{\Upsilon}^a.
\end{aligned} \tag{35}$$

So, the vacuum is defined as

$$\begin{aligned}
\langle 0|b|0\rangle &= \langle 0|i[\tilde{c}, c]|0\rangle - \frac{1}{\alpha} \langle 0|\nabla_a[\gamma] \Upsilon^a|0\rangle, \\
\langle 0|\tilde{b}|0\rangle &= \langle 0|i[\tilde{c}, \tilde{c}]|0\rangle - \frac{1}{\alpha} \langle 0|\tilde{\nabla}_a[\tilde{\gamma}] \tilde{\Upsilon}^a|0\rangle.
\end{aligned} \tag{36}$$

This shows that even in modified MA gauge the ghost-anti-ghost condensates remains present in the ABJM theory and due to which the spontaneous breaking of the BRST symmetry occurs. An advantage of the spontaneous BRST supersymmetry breaking is that the Nambu-Goldstone particle associated with the spontaneous breaking of the BRST symmetry or the spontaneous breaking of anti-BRST symmetry can be identified with the diagonal anti-ghost or diagonal ghost, respectively. Thus, the diagonal ghost and the diagonal anti-ghost are massless. This result is consistent with infrared Abelian dominance. As infrared Abelian dominance is expected to be realized, if the off-diagonal components of ghosts become massive while the diagonal components remain massless.

VI. CONCLUSION

In this paper, we analysed the ABJM theory in Delbourgo-Jarvis and Baulieu-Thierry-Mieg (DJBTM) gauge and modified maximally abelian (MA) gauge. Furthermore, we have investigated the quantum actions for the theory admitting supersymmetric BRST invariance. We have observed that due to ghost-anti-ghost condensates appear in non-linear DJBTM gauge the non-linear bosonic fields admit non-vanishing vacuum expectation values (VEVs). Consequently, a spontaneous breaking of the supersymmetric BRST invariance has occurred in the theory. We also demonstrated that even in modified MA gauge, the ghost-anti-ghost condensates remains present in the ABJM theory, and due to which the spontaneous breaking of the BRST symmetry occurs. We have identified the ghost and anti-ghost fields present in the theory as a Nambu-Goldstone particles according to Nambu-Goldstone theorem. To confirm the appearance of ghost-anti-ghost condensation as well as spontaneous symmetry breaking we constructed an effective potential for the ABJM theory.

It may be noted just as strings can end on D-branes in string theory, M2-branes can also end on M9-branes, M5-branes and gravitational waves in M-theory [36]. Boundary conditions for open M2-branes in presence of a flux have also been studied [37]. A system of multiple M2-branes ending on a M5-brane can be used to learn about the physics of M5-branes. Thus, a system a system of M2-branes ending on a M5-brane with a constant C field in the background has been used to motivate the study of a novel quantum geometry on M5-brane world-volume [38]. In fact, the BLG theory with a Nambu-Poisson 3-bracket has been identified with the M5-brane action in presence of a large C field [39]. The action for M2-branes in presence of a boundary has been constructed in $\mathcal{N} = 1$ superspace formalism [11, 40–42]. This theory is made gauge invariant by adding extra boundary degrees of freedom such that the gauge transformation of the boundary theory exactly cancels the boundary piece generated by gauge transformation of the bulk theory. Similarly, the BRST transformation of the boundary theory exactly cancels by boundary term generated by the BRST transformation of the bulk theory. It will be interesting to investigate what happens to this system, if the BRST symmetry is broken on the boundary or in the bulk due to the ghost-anti-ghost condensation.

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